

INTRODUCING SURROGATE MODELS TO THE STRUCTURAL PRELIMINARY AIRCRAFT DESIGN PHASE

C. Jacob, J. Bieler, A. Bardenhagen
Technical University Berlin, Institute of Aeronautics and Astronautics,
Marchstr. 12, 10587 Berlin, Germany
c.jacob@tu-berlin.de, tel.: +49(0)3031424448

Abstract

During the preliminary aircraft design phase various different configurations with a large number of unknown parameters are analyzed. New methods for preliminary aircraft design need to be implemented in order to meet the requirements of future aircraft. This paper presents an overview of how surrogate models are obtained with the help of the kriging method. Therefore, a short overview of the kriging method is given and a first surrogate model for the structural layout of a primary wing structure is introduced. The primary structure is modeled as a cantilever beam filled with ribs. The number of ribs and the shell thickness are varied within given boundary conditions. The presented surrogate model is based on the results from a finite element model of the given structure. Afterwards, an optimal solution according to the structural weight is found with the help of the surrogate model. These surrogate models will enhance the preliminary design process.

Nomenclature

b	Wing span	W	Aircraft weight
f	Function of which surrogate will be obtained	X	Location in design space
\hat{f}	Surrogate function	x_E	Distance from fuselage to wing-mounted engine
L	Lift	ϵ	Random error
p	Parameter of spatial correlation function	Θ	Parameter of spatial correlation function
W_E	Engine weight	μ	Deterministic trend

1 INTRODUCTION

During a conceptual and preliminary aircraft design a vast number of different configurations with a large number of unknown parameters are analyzed. In order to predict the aircraft weight in early design stages, different methods are used. Elham et al. [1] and Dababneh et al. [2] divide those methods in four different classes. The lower class I and II methods are rather simple handbook methods (see [3], [4]) and mostly based on statistical approaches. Class III methods are based on high fidelity models, which use finite elements methods (FEM, see [5], [6]). The class IV weight estimation methods are only used during the detailed aircraft design phase and not during conceptual or preliminary aircraft design. The statistical methods deliver good results for established aircraft configurations with conventional wing-body configuration and classical combustion engines. The physics based analysis provide very detailed results but consume a lot of calculation and modeling time. Thus, as stated in [2], new methods for early aircraft design need to be implemented in order to meet the requirements of future aircraft.

In the last years approximation methods and optimization based on those became more and more popular. Computation-intensive function are approximated with sim-

ple analytical models of the high fidelity model used in class III or higher methods. The simple models are called surrogate models or metamodels. They help to reduce the number of design variables and their range of search. With those surrogates an optimization can be run and afterwards it can be returned to the higher order model. This can save tremendous calculation time during the design process. [7] [8]

In this paper, a surrogate model of a primary wing structure is presented. Therefore, the method of surrogate modeling with a focus on kriging (see chapter 3.1) is introduced. Emphasis is also given to how the surrogate is obtained and how it can be extended later to an entire aircraft structure. In [9] and [10] the general process of designing a surrogate or metamodel is described as follows:

1. *Design of Experiments (DoE).*

DoE is the sampling plan in the design space. The key question refers to the number of samples considering the limit of the computational cost of the high fidelity model. This process is described in chapter 2, where the high fidelity model of the primary wing structure is introduced as well.

2. *Execution of high-fidelity model at selected location.*

The points determined in the first step are executed in the numerically expensive high fidelity model. In

this paper, a primary wing structure finite element model will be used. A detailed description can be found in chapter 2.

3. Design of the surrogate model.

Given the results of the previous steps, it needs to be decided what kind of surrogate shall be used and how the corresponding parameters are obtained. Chapter 3 describes this process.

4. Model validation.

The last step is to predict the error of the surrogate in comparison to the high fidelity model. Then the quality can be analyzed. That is shown in chapter 4 for the wing structure surrogate.

Afterwards, an optimum in the design space can be found. The given problem will be optimized according to its structural weight. This is described in chapter 4.

2 FINITE ELEMENT MODEL

As previously described a high-fidelity model of a primary wing structure is build to obtain sample points for building a surrogate model. The primary wing structure carries all distributed loads and all concentrated loads as the weights of the fuselage, landing gear and engines. The finite element method is a numerical method which is widely applied to different problems like in solid and structural analysis, thermal analysis, fluid flow analysis, piezoelectric analysis, and many others. The analytical solution is often difficult to obtain. A detailed description can be found in [11].



Figure 1: Dash 8 Q400 [12]

The wing of a Bombardier Dash 8 Q400 (see Figure 1) can be described as a cantilever beam and is used as a basis for the finite element model. The cantilever beam and the applied loads are shown in figure 2. At one end the wing-box is clamped to the wall (left side in figure 3). The total load calculated by the lift (L) necessary to compensate the aircraft weight (W).

It is homogeneously distributed as an area load to two thirds on the upper shell and one third on the lower shell of the beam. Since the load is evenly distributed, the forces will be simplified (e.g. no torsion). The wing-box is filled with ribs (see figure 3). The wing span (b) is given and no tapering or sweep is applied. The whole geometry is meshed with the same shell-thickness using S4 shell elements in Abaqus [13]. Smaller structural elements like

stringer are neglected in this first approach. The weight of the aircraft engine (W_E) is introduced with the help of a rib which is mounted to the wing-box at the distance (x_E) from the fuselage of the aircraft. The outer geometry of the wing-box is fixed but the number of ribs and the shell-thickness are varied. To prevent a dependency of the element size on the outer shell, the ribs are freely placed along the beam and attached with a tie connection. The whole model has the material properties of aluminium 7075-T6.

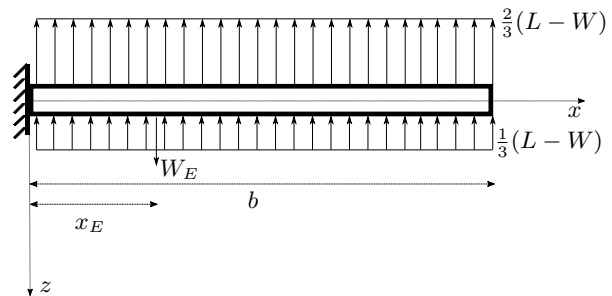


Figure 2: Loads on Primary Wing Structure

The finite element model is kept as simple as possible, so it only covers a stress analysis and stiffness design. In the first approach the strength and strain analysis are neglected.

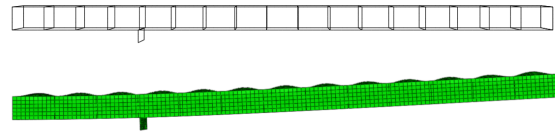


Figure 3: Model as Wireframe and the Deformation

This finite element model of the wing can be used for obtaining results at certain sample points. Those will be used to set up the surrogate model which is described in the next chapter.

3 SURROGATE MODEL

Surrogate models are used in many different engineering disciplines. An overview with examples is given in [8]. Those generated models are cheaper to evaluate than high-fidelity evaluations.

The problem is addressed with obtaining a function $\hat{f}(X)$ of the function f . In [14] a good overview of the possibilities how to obtain a surrogate $\hat{f}(X)$ is given. In [15] the surrogate models are divided in two categories: the interpolation methods and the approximation methods. For the former as examples can be stated the kriging method [7], radial basis functions (RBF) [7] and the scattered data interpolation [16]. For the latter one example is the method of least squares. For optimization and feasibility analysis kriging and RBF surrogates are the most popular choices because of their ability to provide a quantitative error measure in prediction. [14]

Before building the surrogate it is necessary to have input

data available which can either be obtained from a high-fidelity model or from experiments. In this case the data is obtained by running the in chapter 2 described finite element model at certain points. After that kriging will be used as the surrogate model for the high fidelity wing model and an overview of a kriging surrogate is build can be found below in 3.1.

3.1 Kriging

The statistic model kriging is also called gaussian process modeling. The idea was proposed by Daniel Krige 1951 in geostatistics [17] to mine valuation and formally developed by Matheron 1963 [18]. Over the years it became more and more popular in other engineering disciplines (see [14], [19]). The kriging method basically estimates the value of a response at a location based on the known responses of the surrounding points. This technique does not require the manual calibration of parameters which makes it useful, especially if there is no knowledge of the objective function.

In general kriging can be described as:

$$(1) \quad \hat{f}(X) = \mu(X) + \epsilon(X),$$

where $\hat{f}(X)$ is the surrogate, $\mu(X)$ is the deterministic trend and $\epsilon(X)$ is a random error at location X . [14] In the literature different variants of kriging are described:

- *Universal kriging* assumes the model as shown in equation (1). Here $\mu(X)$ is the deterministic trend or function and $\epsilon(X)$ is a random error at location X .

- *Ordinary kriging* formulates the model as

$$(2) \quad \hat{f}(X) = \mu + \epsilon(X),$$

where μ is an unknown constant. As the trend μ is estimated, so is the error ϵ .

- *Simple kriging* formulates the model as

$$(3) \quad \hat{f}(X) = \mu + \epsilon(X),$$

where μ is an exactly known constant. With that also the error ϵ is exactly known at the data locations X .

- *Co-kriging* estimates two different type of models:

$$(4) \quad \hat{f}_1(X) = \mu_1 + \epsilon_1(X),$$

$$(5) \quad \hat{f}_2(X) = \mu_2 + \epsilon_2(X),$$

which is called ordinary co-kriging. The main interest is in $\hat{f}_1(X)$ and it will be correlated with the other variables. Co-kriging can also be performed with more than two variables. The more are used the more calculation time is needed, but the precision can be increased. It can also be used with all other types of kriging.

These are only the most commonly used kinds of kriging. Others are for example indicator kriging, probability kriging or disjunctive kriging. [20]

3.2 Surrogate Model of Wing

Based on the finite element model described in section 2, a surrogate is constructed. With the help of the surrogate the number of ribs and shell-thickness can be optimized according to its minimal structural weight. The boundaries for number of ribs is set to [5 .. 19] and for shell-thickness to [2.0 .. 3.4] mm, based on experience. As described in chapter 1 four major steps have to be performed to obtain a surrogate model. First the design of experiments are done. According to [8] for experiments with systematic errors a space filling sampling plan rather than a boundary concentrated is used. [21] describes a simple algorithm for latin hypercube. With that 14 sample points are determined.

The second step is to run the in chapter 2 obtained high fidelity model of the wing-box at the in the first step selected sample points.

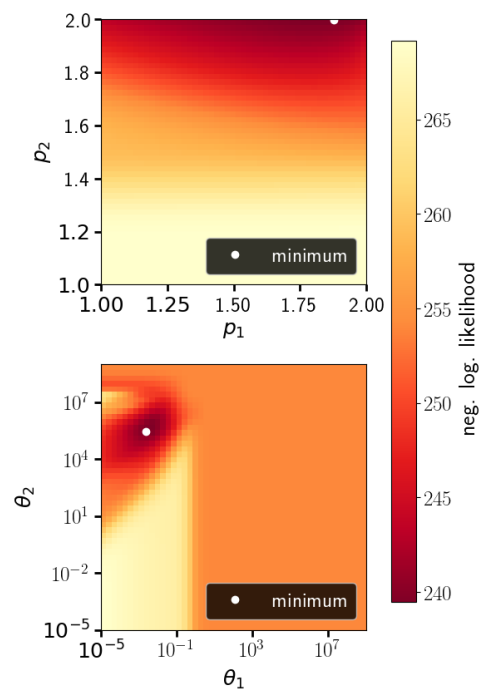


Figure 4: Negative logarithmic likelihood of p_1, p_2 (top) and θ_1, θ_2 (bottom)

The next and third step is to obtain the surrogate. In this case the kriging method, as described in the previous section 3.1 is used. Here ordinary kriging is selected. As described in [7] to fit the model a statistic method is used, where the spatial correlation function between all sampling points is calculated. The spatial correlation is determined using two parameters, the power p and a factor θ . These parameters can be found by minimizing the negative logarithmic likelihood to ensure that the surrogate model is best fitted. The surrogate is constructed in Python, which offers a variety of optimization algorithms to find these variables. However a simple gradient based algorithm showed that it got stuck to often in a local minima. One problem is the big range of the objective variables. While each p lies within the range [1 .. 2], θ accepts values in [$1e^{-5}$.. $1e^{10}$]. Moreover the variables influence each other.

The upper graph in figure 4 shows the negative logarithmic likelihood of the p parameters. θ_1 and θ_2 are constant while p_1 and p_2 are varied. Finding the optimum in this case is easy. A gradient based optimizer like `scipy.minimize` will find the global minima [22]. In the lower graph of Figure 4 θ_1 and θ_2 are varied while p_1 and p_2 are constant. Here can be recognized that the global minima is very close to local maxima which makes the optimization difficult. Basin-hopping of the `scipy` package takes some time, but it combines the variation of random initial values with gradient based optimization which increases the chance of finding the global minimum [22].

The last step, the validation of the model is described in the next chapter 4.

4 RESULTS

In figure 5 the FEM measurements along with the surrogate model and its sample points are visualized.

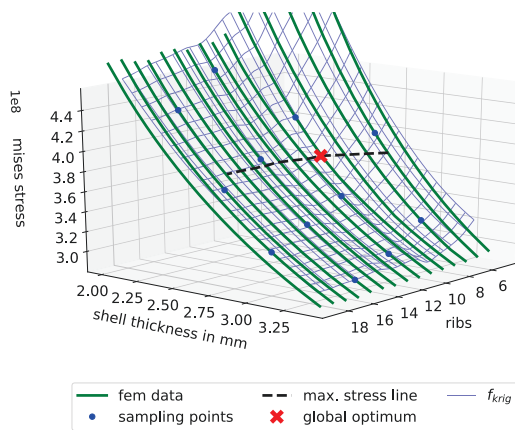


Figure 5: Surrogate Model of the Wing Structure Analysis

The kriging method interpolates the training data and thus the data, that was used to build the surrogate model of the wing can not be used to rate the quality of the results. To avoid this issue, a testing set is generated, in which the high fidelity model is employed. These data are then compared with the results of the surrogate model for the same inputs as the test set as well as compared to each other. [23] As the high-fidelity model is simple and does not require long in order to calculate, more points in the design space are calculated than required to build the surrogate. These points are shown by the green solid lines in figure 5. The difference to the high-fidelity model is around 1% at every point. This shows that the sampling points for the training data are well chosen for building the surrogate. Additionally, the surrogate can be used to further explore the design space and to find fast and accurate solutions for the given problem.

While the FEM results are only available for a natural number of ribs (the shell thickness is linear interpolated between the discrete points), the surrogate model covers continuously the entire design space. A natural number of ribs and a real number for the shell thickness is required to identify the optimal design point. Thus, for every rib in the design range a separate optimization is done to find the minimum shell-thickness that withstands the maximum

stress (connected by a dashed line in fig. 5). For these points, a structural weight can be calculated based on the geometry. Afterwards, the design point with a minimum weight is identified (big cross in fig. 5). In this example the maximum stress of aluminium 7075-T6 is selected, divided by a safety factor of 1.5. The safety-factor is required by CS-25 [24].

5 CONCLUSION AND OUTLOOK

A design engineer would not simply rely on the on the results of a approximation model during a design process. However, surrogate models do give a good first guess where the solution can be found in the design space. [23] Especially in preliminary aircraft design the design space is very large with a many variables. This phenomena is often called “the curse of dimensionality” [7].

However, with the obtained data the designer has the possibility to return to the high fidelity model in order to get more accurate results. Using the surrogate models during the design process, the computational cost can be reduced while exploring large regions of the design space by replacing repeated detailed simulations with the high fidelity model. On the other hand, there can be a substantial computational cost to obtain data from the high fidelity model for building the surrogate models. Thus, the user of the surrogate models needs to carefully analyze if the usage of a surrogate for the design process is an advantage.

In this paper, a first surrogate for a cantilever primary wing structure is presented and therefore an optimum with minimum structural weight is obtained. This presented surrogate model works well to explore the design space further and to find an optimum within the design space. In the future the finite element model needs to be expanded to an entire aircraft structure with different shell thicknesses. Additionally, not only stiffness design needs be covered but also strength and strain analysis in order to cover all effects during loading. The surrogate needs to be updated with the new high-fidelity model. The applied loads need to be expanded towards a flight envelope, which covers load cases from flight, ground, failure and emergency load cases. More inputs variables for the surrogate will be defined as the model will be expanded. These input variables do not only depend on structural conditions like number of ribs but also on outer geometry parameters like wing span. Further, it shall be investigated which kind of kriging methods presented in chapter 3.1, is best for the preliminary aircraft design process. Therefore different surrogate models of the same high-fidelity model will be build. These models will then be compared to each other.

After obtaining a surrogate for an aircraft design space new concepts can be explored. This will enhance the design process of future aircraft and help to understand new technologies. The finite element model optimization will take too long without using surrogate models during the preliminary aircraft design phase. Consequently, surrogate models offer more possibilities to explore a larger design space in less time than high-fidelity models.

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